# January 2001

# Pure Mathematics 30

## Grade 12 Diploma Examination



Alberta
LEARNING

# Pure Mathematics 30 Diploma Examination January, 2001

## Multiple-Choice Key, Numerical-Response Key and Sample Answers to Written-Response Questions

## PURE MATHEMATICS 30 - January, 2001

### MULTIPLE-CHOICE KEY

| | | | |
|---|---|---|---|
| **1.** | A | **18.** | A |
| **2.** | C | **19.** | D |
| **3.** | B | **20.** | A |
| **4.** | B | **21.** | B |
| **5.** | C | **22.** | A |
| **6.** | D | **23.** | D |
| **7.** | B | **24.** | B |
| **8.** | D | **25.** | C |
| **9.** | A | **26.** | B |
| **10.** | B | **27.** | B |
| **11.** | C | **28.** | D |
| **12.** | C | **29.** | A |
| **13.** | A | **30.** | C |
| **14.** | B | **31.** | D |
| **15.** | C | **32.** | C |
| **16.** | D | **33.** | A |
| **17.** | B | | |

### NUMERICAL-RESPONSE KEY

| | |
|---|---|
| **1.** | 8 |
| **2.** | 1.53 |
| **3.** | 210 |
| **4.** | 1.5 |
| **5.** | 0.08 |
| **6.** | 0.02 |

## SAMPLE ANSWERS TO THE WRITTEN-RESPONSE SECTION

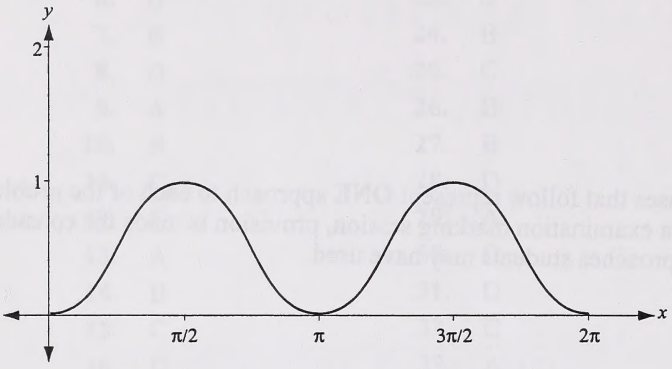
**Note:** The responses that follow represent **ONE** approach to each of the problems. During the diploma examination marking session, provision is made for considering the various approaches students may have used.

## Sample Answer for Written Response 1—Total: 10%

**Written Response—10%**

**1.** • On the graph below, make a sketch of $y = \sin^2 x$ on the domain $0 \le x \le 2\pi$.
You may use your graphing calculator to help you.
**Note:** $\sin^2 x = (\sin x)^2$

**A POSSIBLE SOLUTION**




• What is the range of $y = \sin^2 x$ ?

**A POSSIBLE SOLUTION**

Range is $0 \le y \le 1$.

• Explain how the range of $y = \sin x$ could be used to predict the range of $y = \sin^2 x$.

**A POSSIBLE SOLUTION**

The range of $y = \sin x$ is $-1 \le y \le 1$. For $y = \sin^2 x$ the values of $\sin x$ are squared. Negative and positive $y$ values are positive when squared, therefore the new range is from $0 \le \sin^2 x \le 1$.

- The graph of $y = \sin^2 x$ can also be represented by $y = A\cos\left[B\left(x - \frac{\pi}{2}\right)\right] + \frac{1}{2}$.

  Using the graph you have drawn, determine the values of $A$ and $B$. Justify your answer.

**A POSSIBLE SOLUTION**

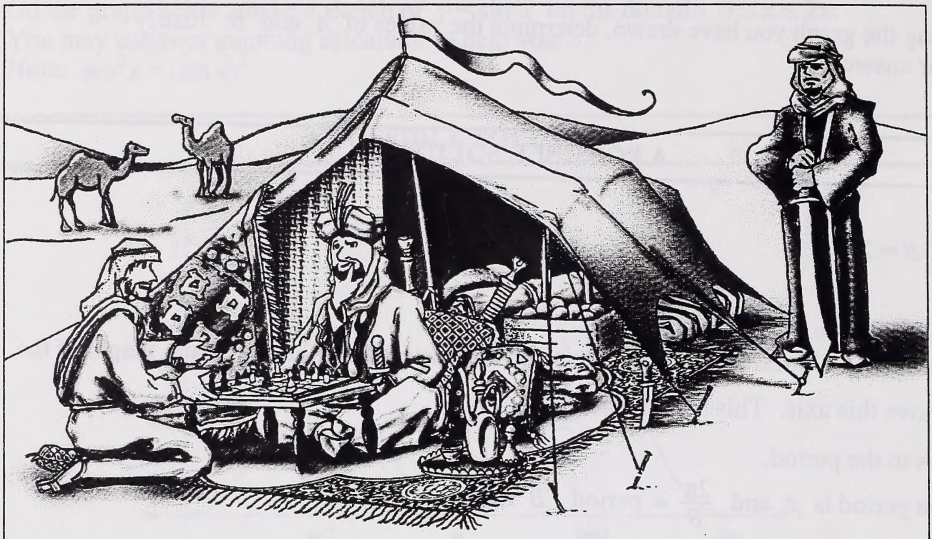
$A = \frac{1}{2}$, $B = 2$

$A$ is the amplitude.
To determine $A$, I located the central axis and then determined the maximum height of the graph above this axis. This height was $\frac{1}{2}$.

$B$ relates to the period.
Since the period is $\pi$ and $\frac{2\pi}{B}$ = period, $B$ must equal 2.

## Sample Answer for Written Response 2—Total: 10%

*Use the following information to answer the next question.*



Legend has it that the game of chess was invented for a Persian king by one of his servants. The king asked the servant how he would like to be paid for the game. The servant stated that he would like one grain of rice to be placed on the first square of the chessboard, two grains of rice to be placed on the second square, four grains of rice on the third square, eight grains of rice on the fourth square, and so on. Each subsequent square was to have twice as many grains of rice than the previous square, as shown in the chart below.

| **Square** | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| **Grain of rice in the square** | 1 | 2 | 4 | 8 | | | |

**Written Response—10%**

**2.** • Complete the chart for squares 5, 6, and 7.

**A POSSIBLE SOLUTION**

| **Square** | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| **Grain of rice in the square** | 1 | 2 | 4 | 8 | **16** | **32** | **64** |

- Write an expression that represents the numbers of grains of rice, $R$, on the $n^{th}$ square of the chessboard.

**A POSSIBLE SOLUTION**

$R = 2^{n-1}$ or $R = \frac{1}{2}(2^n)$ where $R$ is the number of rice grains and $n$ is the square number.

- If the king only had 1 000 000 grains of rice, which square would he be in the process of filling when he ran out of rice?

**A POSSIBLE SOLUTION**

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$1\,000\,000 = \frac{1(2^n - 1)}{2 - 1}$$

$$1\,000\,001 = 2^n$$

$$n = 19.93$$

$\therefore$ in the $20^{th}$ square

- If the servant had asked for the payment of rice to be placed on only the black squares of the chessboard, with 1 grain on the first black square, 2 grains on the second black square, 4 grains on the third black square, and so on, then only 32 squares would have rice on them. What would be the **total** number of grains of rice on the chessboard if the king filled every black square?

**A POSSIBLE SOLUTION**

1, 2, 4, ...

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_{32} = \frac{1(2^{32} - 1)}{2 - 1}$$

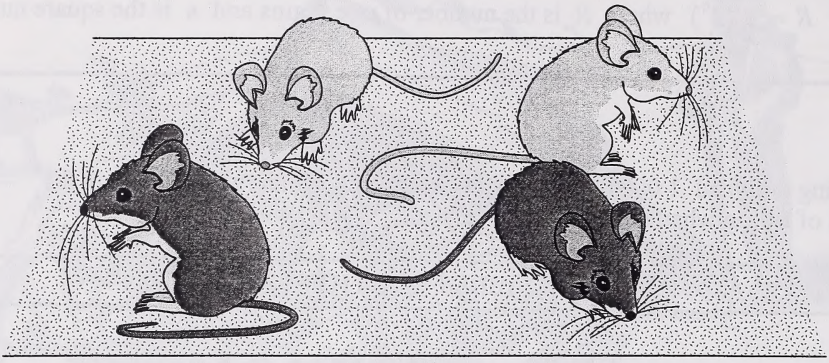
$$S_{32} = 4\,294\,967\,295$$

There are 4 294 967 295 grains of rice.

## Sample Answer for Written Response 3—Total: 10%

*Use the following information to answer the next question.*

The breeding of yellow mice produces offspring that are either yellow or grey.



A student used Y to indicate a yellow mouse and G to indicate a grey mouse.

**Written Response—15%**

3. • The student wrote YYGG to show the outcome of a breeding that produced 4 offspring. List all other outcomes that could exist for the 4 offspring, if order is not important.

| A POSSIBLE SOLUTION |
|---|
| G G G G<br>G G G Y<br>G Y Y Y<br>Y Y Y Y<br>There are 4 other outcomes. |

*Use the following information to answer the next part of the question.*

Yellow mice are mutants and their breeding produces offspring that may not survive as a result of the mutation. Of the offspring that do survive, $P(\text{Grey}) = \frac{1}{3}$ and $P(\text{yellow}) = \frac{2}{3}$.

- Determine the probability, to four decimal places, of the last two of the following outcomes.

The probability that 0 out of 4 survivors are grey is 0.1975.
The probability that 1 out of 4 survivors is grey is 0.3951.
The probability that 2 out of 4 survivors are grey is 0.2963.
The probability that 3 out of 4 survivors are grey is _____.
The probability that 4 out of 4 survivors are grey is _____.

**A POSSIBLE SOLUTION**

$$P(0) = {}_4C_0\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^4 = 0.1975$$

$$P(1) = {}_4C_1\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^3 = 0.3951$$

$$P(2) = {}_4C_2\left(\frac{2}{3}\right)^2\left(\frac{2}{3}\right)^2 = 0.2963$$

$$P(3) = {}_4C_3\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^1 = \mathbf{0.0988}$$

$$P(4) = {}_4C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^0 = \mathbf{0.0123}$$

- The sum of all the probabilities in the previous bullet equals one. Explain why the sum of all the probabilities will be the same for any number of offspring.

**A POSSIBLE SOLUTION**

The sum of sample space probabilities is 1.

No matter how many offspring mice, the sum of all possible events in the sample space must equal 1, to account for all possibilities.

- Determine the minimum value of $n$ in order for this distribution to be considered a "large sample" so that it could be approximated by the normal distribution curve.

**A POSSIBLE SOLUTION**

$$n(p) \geq 5 \quad \text{and} \quad n(1-p) \geq 5$$

$$n\left(\frac{1}{3}\right) \geq 5 \qquad n\left(\frac{2}{3}\right) \geq 5$$

$$n \geq 15 \quad \text{and} \quad n \geq 7.5$$

$$\therefore\ n = 15 \text{ minimum}$$

- If several pairs of yellow mice are bred and 210 offspring survive, what is the probability, to the nearest hundredth, that at most 80 of the 210 offspring are grey?

**A POSSIBLE SOLUTION**

$210\left(\frac{1}{3}\right) = 70 \geq 5$

$210\left(\frac{2}{3}\right) = 140 \geq 5 \quad \therefore$ Large sample

$\mu = np = 210\left(\frac{1}{3}\right) = 70$

$\sigma = \sqrt{np(1-p)} = 6.83$

$z = \frac{80 - 70}{6.83} = 1.46$

$P(z \leq 1.46) = 0.9279$
$= 0.93$

normalcdf (–1E99, 80, 70, 6.83)

0.9284205…

$P$(at most 80) = 0.93

Binompdf $\left(210, \frac{1}{3}, \text{seq}(x, x, 81, 210, 1)\right)$

1 – sum(ans) = 0.93664…
= 0.94

*January 2001*

# *Pure Mathematics 30*

## *Grade 12 Diploma Examination*

### *Description*

**Time**: This examination was developed to be completed in 2.5 h; however, you may take an additional 0.5 h to complete the examination.

This is a **closed-book** examination consisting of

- 33 multiple-choice and 6 numerical-response questions, of equal value, worth 65% of the examination
- 3 written-response questions worth 35% of the examination

A tear-out formula sheet and two $z$-score pages are included in this booklet.

All graphs on this examination are computer-generated.

***Note:*** *The perforated pages at the back of this booklet may be torn out and used for your rough work.* ***No marks*** *will be given for work done on the tear-out pages.*

### *Instructions*

- You are expected to provide a graphing calculator approved by Alberta Learning.
- You are expected to have cleared your calculator of all information that is stored in the programmable or parametric memory.
- Use only an HB pencil for the machine-scored answer sheet.
- Fill in the information required on the answer sheet and the examination booklet as directed by the presiding examiner.
- Read each question carefully.
- Consider all numbers used in the questions to be **exact** numbers and not the result of a measurement.
- If you wish to change an answer, erase **all** traces of your first answer.
- Do not fold the answer sheet.
- The presiding examiner will collect your answer sheet and examination booklet and send them to Alberta Learning.
- Now turn this page and read the detailed instructions for answering machine-scored and written-response questions.

## *Multiple Choice*

- Decide which of the choices **best** completes the statement or answers the question.
- Locate that question number on the separate answer sheet provided and fill in the circle that corresponds to your choice.

***Example***

This examination is for the subject of

**A.** biology
**B.** physics
**C.** chemistry
**D.** mathematics

Answer Sheet

Ⓐ Ⓑ Ⓒ ●

## *Numerical Response*

- Record your answer on the answer sheet provided by writing it in the boxes and then filling in the corresponding circles.
- If an answer is a value between 0 and 1 (e.g., 0.7), then be sure to record the 0 before the decimal place.
- **Enter the first digit of your answer in the left-hand box and leave any unused boxes blank.**

***Examples***

***Calculation Question and Solution***

Correct to the nearest tenth of a radian, $40^\circ$ is equal to _____ rad.

$40^\circ = 0.6981317008\ldots$ rad
$\doteq 0.7$

(Record your answer in the numerical-response section on the answer sheet.)

**Record 0.7 on the answer sheet** →

### *Correct-Order Question and Solution*

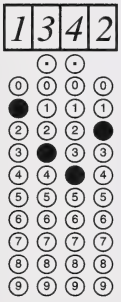
When the following subjects are arranged in alphabetical order, the order is _____, _____, _____, and _____.

**1** biology
**2** physics
**3** chemistry
**4** mathematics

(Record **all four digits** of your answer in the numerical-response section on the answer sheet.)

Answer: 1342

**Record 1342 on the answer sheet** →

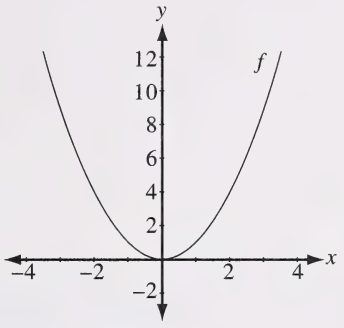



### *Written Response*

- Write your answers in the examination booklet as neatly as possible.
- For full marks, your answers must address **all** aspects of the question.
- Descriptions and/or explanations of concepts must be correct and include pertinent ideas, diagrams, calculations, and formulas.
- Your answers must be presented in a well-organized manner using complete sentences and correct units.

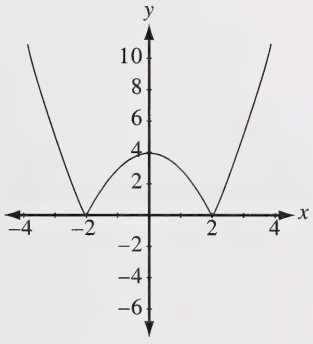
*Use the following information to answer the first question.*

The graph of $y = f(x)$ is shown below.

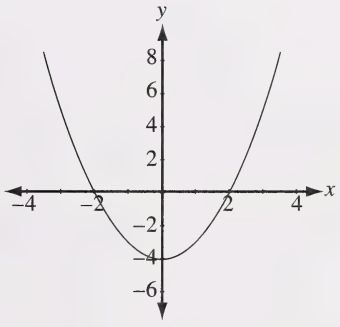



**1.** The graph of $y = |f(x) - 4|$ is

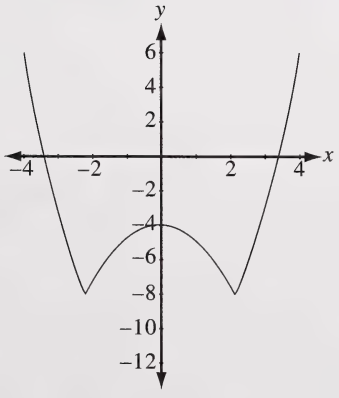
**A.**



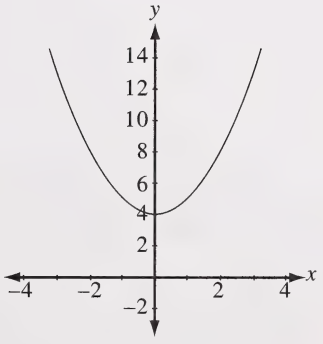

**B.**




**C.**




**D.**



**2.** If $f(x) = 2x^2 - 3$, where $x \geq 0$, then a function $g$ that will have a domain and range that are **both** different from those of function $f$ is

**A.** $g(x) = f(-x)$

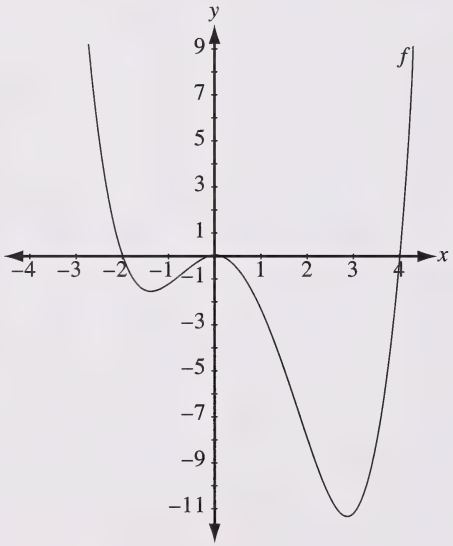
**B.** $g(x) = -f(x)$

**C.** $g(x) = f^{-1}(x)$

**D.** $g(x) = kf(x),\ k > 0$

*Use the following information to answer the next question.*

The partial graph of the function $y = f(x)$ is shown below. The range of function $f$ is $f(x) \geq -11$.




**3.** If function $f$ is transformed to a new function $g(x) = f(x - 3) + 2$, then the range of function $g$ will be

**A.** $g(x) \geq -11$

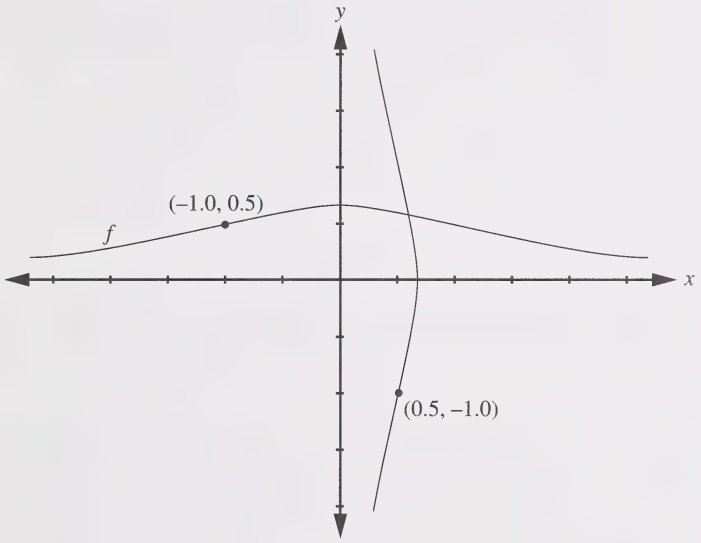
**B.** $g(x) \geq -9$

**C.** $g(x) \geq -8$

**D.** $g(x) \geq 0$

*Use the following information to answer the next question.*

The partial graph of the function $f(x) = \dfrac{2}{x^2 + 3}$ and a reflection of it are shown below.




**4.** An expression for this reflection of graph $f$ is

**A.** $x = y$

**B.** $x = f(y)$

**C.** $y = f(-x)$

**D.** $y = -f(x)$

## Numerical Response

**1.** The graph of $y = f(x)$, where $f(x) = |x + 4| + 1$, is reflected in the $y$-axis. This produces the same results as would translating the graph of $y = f(x)$ to the right by _____ units.

(Record your answer in the numerical-response section on the answer sheet.)

**5.** What is one-half of $2^{20}$ ?

**A.** $2^{10}$

**B.** $1^{20}$

**C.** $2^{19}$

**D.** $1^{10}$

**6.** If $\log_3 y = c - \log_3 x$, where $y > 0$ and $x > 0$, then $y$ is equal to

**A.** $c - x$

**B.** $\frac{c}{x}$

**C.** $\frac{c^3}{x}$

**D.** $\frac{3^c}{x}$

**7.** The sum of the infinite geometric series $\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots$ is

**A.** $\frac{21}{64}$

**B.** $\frac{1}{3}$

**C.** $\frac{1}{2}$

**D.** 1

**8.** An investment of \$1 000 is earning 4% interest per annum compounded annually. If the value, $V$, of the investment after $t$ years is given by $V = 1\,000(1.04)^t$, then $t$, written as a function of $V$, is

**A.** $t = \frac{\log(V)}{3} - \log(1.04)$

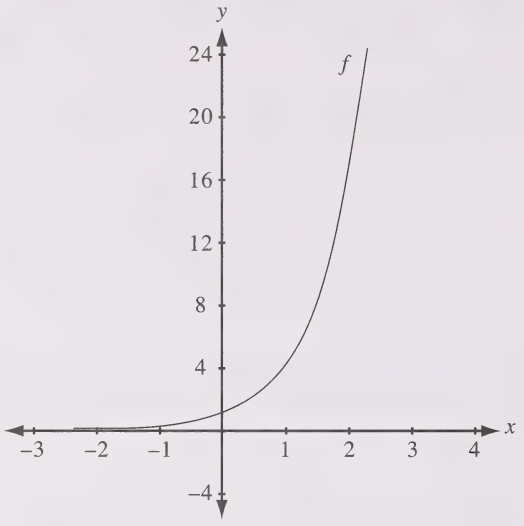
**B.** $t = \frac{\log(V)}{3\log(1.04)}$

**C.** $t = \log(V) - 3 - \log(1.04)$

**D.** $t = \frac{\log(V) - 3}{\log(1.04)}$

*Use the following information to answer the next question.*

The partial graph of the exponential function $f(x) = 4^x$ is shown below.




**9.** The domain of the inverse function $f^{-1}$ is

**A.** $x > 0,\ x \in R$

**B.** $x < 0,\ x \in R$

**C.** $x \geq 0,\ x \in R$

**D.** $x \in R$

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**10.** The population of a city was 173 500 on January 1, 1978, and it was 294 000 on January 1, 1992. If the growth rate of the city can be modelled as an exponential function, then the average annual growth rate of the city, expressed to the nearest tenth of a percentage, was

**A.** 1.0%

**B.** 3.8%

**C.** 6.9%

**D.** 12.1%

**11.** The price of a particular product doubles every 35 years. If the price of the product was \$16.40 on January 1, 1996, then the price of the product will be \$36.50 in the year

**A.** 2028

**B.** 2031

**C.** 2036

**D.** 2040

*Use the following information to answer the next question.*

The solution to a particular problem is the intersection point of two exponential functions, $y = \left(\frac{1}{2}\right)^{x-1}$ and $y = 3^x$.

**Numerical Response**

**2.** Using a graphing calculator, a student found that the $y$-coordinate of this point, correct to the nearest hundredth, was _____.

(Record your answer in the numerical-response section on the answer sheet.)

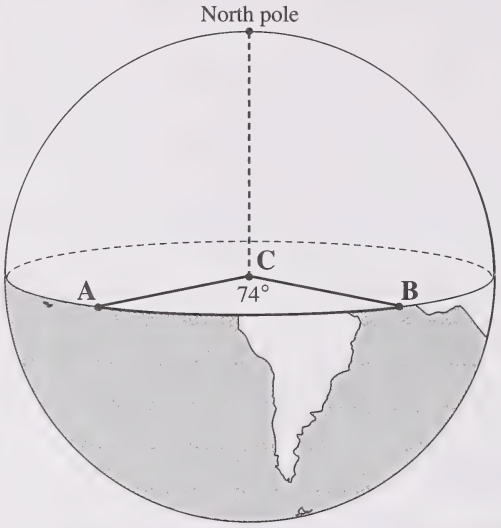
**Numerical Response**

**3.** If point $P\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ lies on the terminal arm of a rotation angle, then the measure of the smallest positive angle, in standard position, to the nearest degree, is _____°.

(Record your answer in the numerical-response section on the answer sheet.)

*Use the following information to answer the next question.*

Two points, $A$ and $B$, are on Earth's equator, and point $C$ is at the centre of Earth. The measure of $\angle ACB$ is 74°, as shown below.




**12.** If the circumference of Earth at the equator is approximately 40 070 km, then the shortest arc length from point $A$ to point $B$, correct to the nearest kilometre, is

**A.** 31 026 km

**B.** 16 474 km

**C.** 8 237 km

**D.** 4 938 km

**13.** The expression $\dfrac{1+\cot\theta}{\csc\theta}$ is equivalent to

**A.** $\sin\theta + \cos\theta$

**B.** $\dfrac{\sin\theta + \cos\theta}{\sin\theta}$

**C.** $\dfrac{1+\cos\theta}{\sin\theta}$

**D.** $1 + \cos\theta$

**14.** If the value of $\cot A$ and the value of $\sec A$ are both negative, then

**A.** $0 < A < \dfrac{\pi}{2}$

**B.** $\dfrac{\pi}{2} < A < \pi$

**C.** $\pi < A < \dfrac{3\pi}{2}$

**D.** $\dfrac{3\pi}{2} < A < 2\pi$

**15.** Given that the function $f(\theta) = a\sin(b\theta)$ has exactly 5 zeros for $0 \le \theta \le 2\pi$, it is possible to determine that

**A.** $a = 5$

**B.** $a = 10$

**C.** $b = 2$

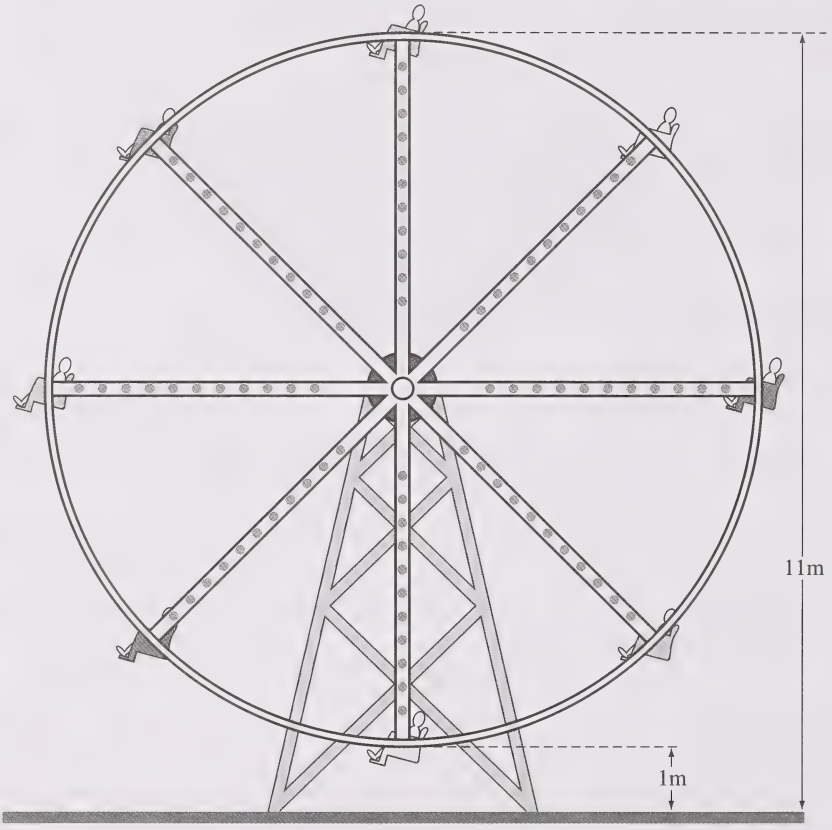
**D.** $b = 2.5$

*Use the following information to answer the next question.*

The distance above the ground of a passenger on a circular ferris wheel is given by the equation

$$h(t) = 5\sin\left[\frac{\pi}{12}(t - 6)\right] + 6$$

where $h$ is the distance above the ground, in metres, and $t$ is the time, in seconds, after the passenger passes the lowest point of the ride for the first time.




**16.** The distance of the passenger above the ground 10 s after passing the lowest point of the ride, to the nearest tenth of a metre, is

**A.** 4.6 m

**B.** 6.1 m

**C.** 8.5 m

**D.** 10.3 m

*Use the following information to answer the next question.*

Refraction describes the bending of light rays. Refraction can be calculated using the formula

$$n = \frac{\sin(\theta + \alpha)}{\sin \theta},$$

where $n$ represents refraction.

**17.** If $\alpha = 30°$, then an equivalent expression for $n$ is

**A.** $\frac{\sqrt{3}}{2} + \cos\theta$

**B.** $\frac{\sqrt{3}}{2} + \frac{1}{2}\cot\theta$

**C.** $\frac{\sqrt{3}}{2} + \frac{1}{2}\cos\theta$

**D.** $\frac{1}{2} + \frac{\sqrt{3}}{2}\cot\theta$

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**18.** If the period of $f(x) = \sin(kx)$, where $k \neq 0$, is $A$ and the period of $g(x) = \left|\sin(kx)\right|$, where $k \neq 0$, is $B$, then

**A.** $A = 2B$

**B.** $A = \frac{1}{2}B$

**C.** $A = \pi B$

**D.** $A = B$

**19.** A plane is parallel to the axis of a double-napped cone. The two possible conic sections formed by the intersection of the cone and the plane are

**A.** a point or a circle

**B.** a line or a parabola

**C.** a point or an ellipse

**D.** intersecting lines or a hyperbola

**20.** In the relation $\frac{(x-1)^2}{a^2} - \frac{(y+2)^2}{16} = 1,\ x \leq -4$ or $x \geq 6,$ the value of $a$ is

**A.** 5

**B.** 10

**C.** 25

**D.** 100

**21.** If the equation of a particular ellipse is $3x^2 + y^2 - 12x + 10y = 0,$ then the centre of this ellipse is point

**A.** (6, 5)

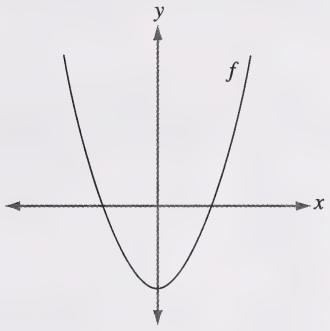
**B.** (2, –5)

**C.** (–6, 5)

**D.** (–2, –5)

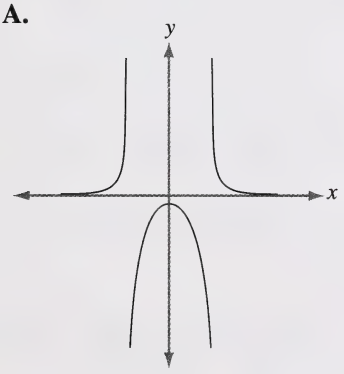
*Use the following information to answer the next question.*

The partial graph of $y = f(x)$ is shown below.

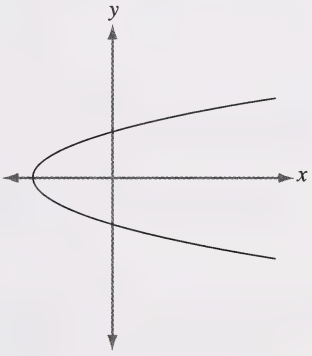



**22.** The partial graph of $y = \dfrac{1}{f(x)}$ is

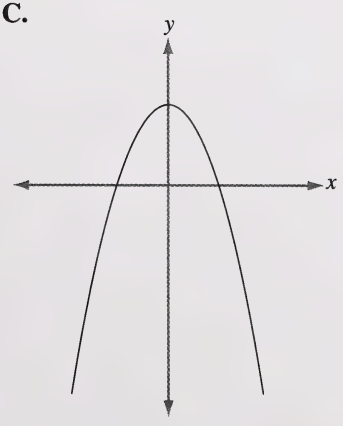
**A.**



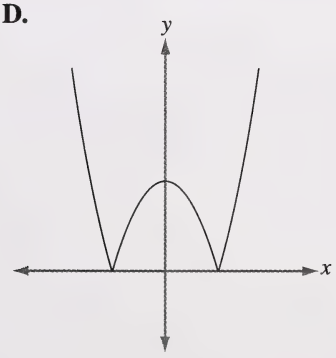

**B.**




**C.**




**D.**



*Use the following information to answer the next question.*

At a company picnic, employees used their cooking skills in a chili cook-off. Employees were required to select the specified number of items from each of the following lists.

| **List A** (select 1) | **List B** (select 2) | **List C** (select 3) | **List D** (select 2) | **List E** (select 2) |
|---|---|---|---|---|
| • 250 g pork<br>• 250 g beef | • 375 mL tomato sauce<br>• 375 mL salsa<br>• 375 mL mixture of water and ketchup | • 1/2 cup onion<br>• 1/4 cup green pepper<br>• 1/4 cup red pepper<br>• 1 cup mushrooms | • 375 mL kidney beans<br>• 375 mL pork and beans<br>• 375 mL lima beans | • 1 tbsp seasoning salt<br>• 1 tbsp garlic salt<br>• 2 tbsp chili powder<br>• 2 tbsp hot sauce |

**23.** The number of different chili recipes that were possible was

**A.** 18

**B.** 120

**C.** 144

**D.** 432

---

**24.** If all of the letters in the word **DIPLOMA** are used, then the number of different 7-letter arrangements that can be made beginning with 3 vowels is

**A.** 24

**B.** 144

**C.** 720

**D.** 5 040

*Use the following information to answer the next question.*

Library books at Grande Prairie Regional College all have bar codes. The bar codes have 14 digits, and the first 8 digits are always 3 1847 000. The remaining 6 spaces can be filled by any digits. An example is shown below.


**25.** The number of different bar codes available for books at this library is

**A.** 6!

**B.** 10!

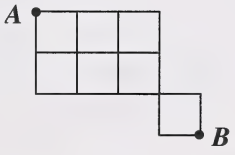
**C.** $10^6$

**D.** ${}_{10}P_6$

---

*Use the following information to answer the next question.*

A student must draw a path from point $A$ to point $B$ in the diagram below.




**26.** If each path must be drawn along the lines such that it is always getting closer to $B$, then the number of paths that the student can draw is

**A.** 12

**B.** 20

**C.** 40

**D.** 70

**Numerical Response**

**4.** If one term in the expansion of $(x-b)^{10}$, $b>0$, is $\frac{76\,545}{32}x^4$, then the value of $b$, correct to the nearest tenth, is _____.

(Record your answer in the numerical-response section on the answer sheet.)

**27.** If a set of data has a standard deviation of 0, then

**A.** the mean of the data must be 0

**B.** all of the data values are the same

**C.** the data values collected had a sum of 0

**D.** the $z$-score of the mean of the data is equal to 1

**28.** On a test, the marks were normally distributed with a mean mark of 54 and a standard deviation of 9. If the number of students who received a mark less than or equal to $k$ was twice the number of those who received a mark of 80 or better, then the value of $k$, correct to the nearest whole number, was

**A.** 52

**B.** 48

**C.** 41

**D.** 30

*Use the following information to answer the next question.*

At a concert, a random sample of ticket buyers revealed that the amount of time they had waited in line to purchase their tickets was normally distributed with a mean of 185 minutes and a standard deviation of 15 minutes.

**29.** The symmetric 95% confidence interval for mean waiting time, in minutes, for all ticket buyers would be approximately

**A.** $155 < \mu < 215$

**B.** $160 < \mu < 210$

**C.** $176 < \mu < 206$

**D.** $182 < \mu < 188$

---

**Numerical Response**

**5.** In a particular town, 70% of the students are bused to school. If a statistician randomly sampled 1 000 students from the town, then the probability that at least 720 of these students are bused to school, correct to the nearest hundredth, would be _____.

(Record your answer in the numerical-response section on the answer sheet.)

**30.** A certain soccer player has scored on 82% of his penalty kicks throughout his career. Given this information, the probability that he will score on exactly 4 of his next 5 penalty kicks, correct to the nearest hundredth, is

**A.** 0.80

**B.** 0.66

**C.** 0.41

**D.** 0.08

*Use the following information to answer the next question.*

A green die has 6 sides, where two sides are labelled with a 1, two sides are labelled with a 2, and two sides are labelled with a 3. A red die has 6 sides. Each side of the red die is labelled with one of the numbers 1, 2, 3, 4, 5, and 6, with each number used only once.

**31.** The sample space for rolling each of these dice once would not include rolling a

**A.** 2 on each die

**B.** 1 on the green die and a 3 on the red die

**C.** 2 on the green die and a 5 on the red die

**D.** 4 on the green die and a 6 on the red die

---

**32.** If $P(A) = \frac{3}{4}$ and $P(A \text{ and } B) = \frac{1}{2}$, where $A$ and $B$ are dependent events, then $P(B \mid A)$ equals

**A.** $\frac{1}{4}$

**B.** $\frac{3}{8}$

**C.** $\frac{2}{3}$

**D.** $\frac{5}{4}$

*Use the following information to answer the next question.*

Peter places the 5 equal-sized tiles shown below in a cloth bag.


**Numerical Response**

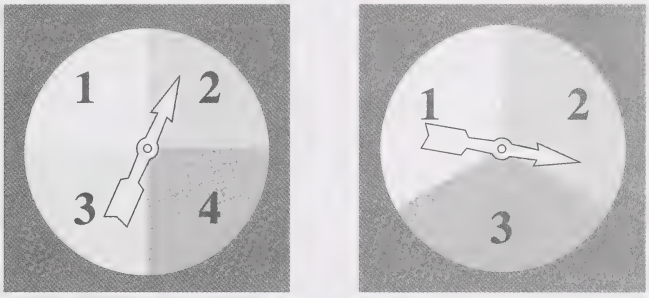
**6.** The probability that Peter selects the 5 tiles, one at a time, in order such that they spell **PETER**, correct to the nearest hundredth, is _____.

(Record your answer in the numerical-response section on the answer sheet.)

*Use the following information to answer the next question.*

**Probability of Independent Events**

Two spinners are shown below. One has four equal-sized areas, and the other has three equal-sized areas. Assume that each spinner is spun once and that each arrow stops in one of the numbered areas.




**33.** The probability that the sum of the numbers indicated by the arrows is an even number is

**A.** $\frac{1}{2}$

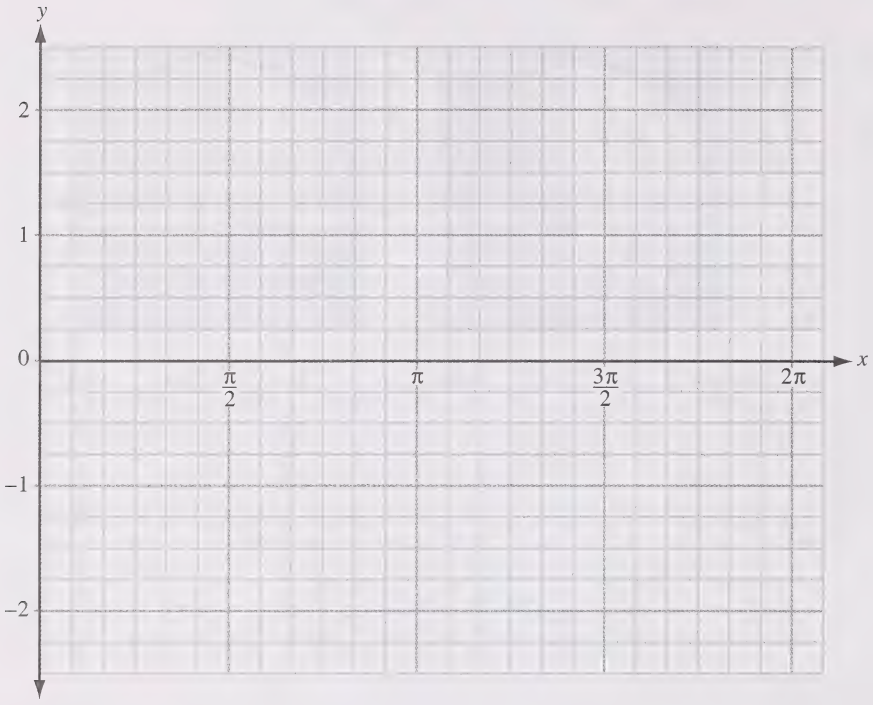
**B.** $\frac{3}{7}$

**C.** $\frac{1}{6}$

**D.** $\frac{2}{3}$

**Written Response—10%**

**1.** • On the graph below, make a sketch of $y = \sin^2 x$ on the domain $0 \leq x \leq 2\pi$.
You may use your graphing calculator to help you.
**Note:** $\sin^2 x = (\sin x)^2$




• What is the range of $y = \sin^2 x$ ?

- Explain how the range of $y = \sin x$ could be used to predict the range of $y = \sin^2 x$.

- The graph of $y = \sin^2 x$ can also be represented by $y = A\cos\left[B\left(x - \frac{\pi}{2}\right)\right] + \frac{1}{2}$. Using the graph you have drawn, determine the values of $A$ and $B$. Justify your answer.

*Written-response question 2 begins on the next page.*

*Use the following information to answer the next question.*

Legend has it that the game of chess was invented for a Persian king by one of his servants. The king asked the servant how he would like to be paid for the game. The servant stated that he would like one grain of rice to be placed on the first square of the chessboard, two grains of rice to be placed on the second square, four grains of rice on the third square, eight grains of rice on the fourth square, and so on. Each subsequent square was to have twice as many grains of rice than the previous square, as shown in the chart below.

| **Square** | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| **Number of grains of rice in the square** | 1 | 2 | 4 | 8 | | | |

**Written Response—10%**

**2.** • Complete the chart for squares 5, 6, and 7.

• Write an expression that represents the numbers of grains of rice, $R$, on the $n^{th}$ square of the chessboard.

- If the king only had 1 000 000 grains of rice, which square would he be in the process of filling when he ran out of rice?

- If the servant had asked for the payment of rice to be placed on only the black squares of the chessboard, with 1 grain on the first black square, 2 grains on the second black square, 4 grains on the third black square, and so on, then only 32 squares would have rice on them. What would be the **total** number of grains of rice on the chessboard if the king filled every black square?

*Written-response question 3 begins on the next page.*

*Use the following information to answer the next question.*

The breeding of yellow mice produces offspring that are either yellow or grey.

A student used Y to indicate a yellow mouse and G to indicate a grey mouse.

**Written Response—15%**

**3.** • The student wrote YYGG to show the outcome of a breeding that produced 4 offspring. List all other outcomes that could exist for the 4 offspring, if order is not important.

*Use the following information to answer the next part of the question.*

Yellow mice are mutants and their breeding produces offspring that may not survive as a result of the mutation. Of the offspring that do survive, $P(\text{Grey}) = \frac{1}{3}$ and $P(\text{yellow}) = \frac{2}{3}$.

- Determine the probability, to four decimal places, of the last two of the following outcomes.

  The probability that 0 out of 4 survivors are grey is 0.1975.

  The probability that 1 out of 4 survivors is grey is 0.3951.

  The probability that 2 out of 4 survivors are grey is 0.2963.

  The probability that 3 out of 4 survivors are grey is _____ .

  The probability that 4 out of 4 survivors are grey is _____ .

- The sum of all the probabilities in the previous bullet equals one. Explain why the sum of all the probabilities will be the same for any number of offspring.

*Written-response question 3 continues on the next page.*

- Determine the minimum value of $n$ in order for this distribution to be considered a "large sample" so that it could be approximated by the normal distribution curve.

- If several pairs of yellow mice are bred and 210 offspring survive, what is the probability, to the nearest hundredth, that at most 80 of the 210 offspring are grey?

***You have now completed the examination.***
***If you have time, you may wish to check your answers.***

# *Pure Mathematics 30 Formula Sheet*

The following information may be useful in writing this examination.

For $ax^2 + bx + c = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

## Exponents, Logarithms, and Geometric Series

$$\log_a(M \times N) = \log_a M + \log_a N$$

$$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$$

$$\log_a M^n = n \log_a M$$

$$\log_b c = \frac{\log_a c}{\log_a b}$$

$$t_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1},\ r \neq 1$$

$$S_n = \frac{rt_n - a}{r - 1},\ r \neq 1$$

$$S = \frac{a}{1 - r},\ |r| < 1$$

## Conics

*General Form*

$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$

*Standard Form*

$$\frac{(x - h)^2}{a^2} \pm \frac{(y - k)^2}{b^2} = \pm 1$$

$$y - k = a(x - h)^2$$

$$x - h = a(y - k)^2$$

## Permutations and Combinations

$${}_nP_r = \frac{n!}{(n - r)!}$$

$${}_nC_r = \frac{n!}{(n - r)!\,r!}$$

In the expansion of $(x + y)^n$, the general term is $t_{k+1} = {}_nC_k\ x^{n-k}y^k$.

Graphing Calculator Window Format

$x$: $[x_{min},\ x_{max},\ x_{scl}]$

$y$: $[y_{min},\ y_{max},\ y_{scl}]$

## Statistics

$$\mu = np \qquad \sigma = \sqrt{np(1 - p)}$$

$$z = \frac{x - \mu}{\sigma}$$

If $np \geq 5$ **and** $n(1 - p) \geq 5$, then the binomial distribution is a large sample.

## Probability

$$P(A \text{ and } B) = P(A) \times P(B/A)$$

$$P(k) = {}_nC_k\ p^k(1 - p)^{n-k}$$

## Trigonometry

$$a = r\theta$$

$$\csc x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

$$\sin(A + B) = \sin A \cos B + \sin B \cos A$$

$$\sin(A - B) = \sin A \cos B - \sin B \cos A$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

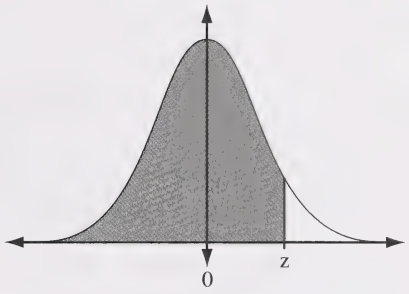
$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\sin(2A) = 2 \sin A \cos A$$

$$\cos(2A) = \cos^2 A - \sin^2 A$$

*Fold and tear along perforation.*

$$z = \frac{x - \mu}{\sigma}$$



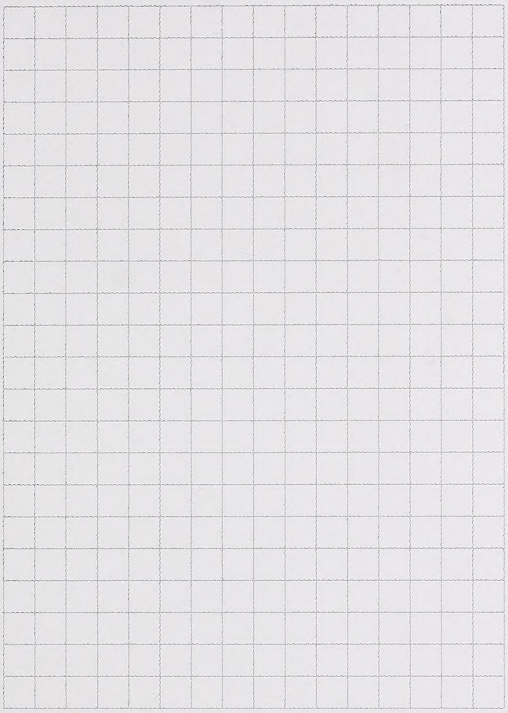

***Areas under the Standard Normal Curve***

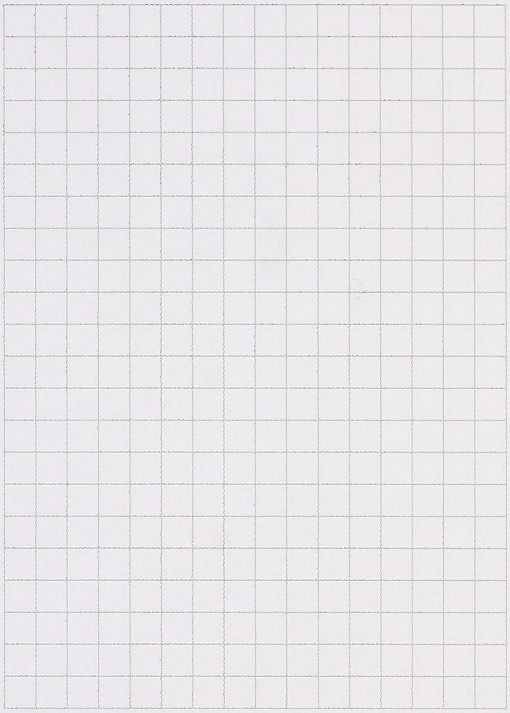
| $z$ | **0.09** | **0.08** | **0.07** | **0.06** | **0.05** | **0.04** | **0.03** | **0.02** | **0.01** | **0.00** |
|---|---|---|---|---|---|---|---|---|---|---|
| **–3.4** | 0.0002 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 |
| **–3.3** | 0.0003 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0005 | 0.0005 | 0.0005 |
| **–3.2** | 0.0005 | 0.0005 | 0.0005 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0007 | 0.0007 |
| **–3.1** | 0.0007 | 0.0007 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0009 | 0.0009 | 0.0009 | 0.0010 |
| **–3.0** | 0.0010 | 0.0010 | 0.0011 | 0.0011 | 0.0011 | 0.0012 | 0.0012 | 0.0013 | 0.0013 | 0.0013 |
| **–2.9** | 0.0014 | 0.0014 | 0.0015 | 0.0015 | 0.0016 | 0.0016 | 0.0017 | 0.0018 | 0.0018 | 0.0019 |
| **–2.8** | 0.0019 | 0.0020 | 0.0021 | 0.0021 | 0.0022 | 0.0023 | 0.0023 | 0.0024 | 0.0025 | 0.0026 |
| **–2.7** | 0.0026 | 0.0027 | 0.0028 | 0.0029 | 0.0030 | 0.0031 | 0.0032 | 0.0033 | 0.0034 | 0.0035 |
| **–2.6** | 0.0036 | 0.0037 | 0.0038 | 0.0039 | 0.0040 | 0.0041 | 0.0043 | 0.0044 | 0.0045 | 0.0047 |
| **–2.5** | 0.0048 | 0.0049 | 0.0051 | 0.0052 | 0.0054 | 0.0055 | 0.0057 | 0.0059 | 0.0060 | 0.0062 |
| **–2.4** | 0.0064 | 0.0066 | 0.0068 | 0.0069 | 0.0071 | 0.0073 | 0.0075 | 0.0078 | 0.0080 | 0.0082 |
| **–2.3** | 0.0084 | 0.0087 | 0.0089 | 0.0091 | 0.0094 | 0.0096 | 0.0099 | 0.0102 | 0.0104 | 0.0107 |
| **–2.2** | 0.0110 | 0.0113 | 0.0116 | 0.0119 | 0.0122 | 0.0125 | 0.0129 | 0.0132 | 0.0136 | 0.0139 |
| **–2.1** | 0.0143 | 0.0146 | 0.0150 | 0.0154 | 0.0158 | 0.0162 | 0.0166 | 0.0170 | 0.0174 | 0.0179 |
| **–2.0** | 0.0183 | 0.0188 | 0.0192 | 0.0197 | 0.0202 | 0.0207 | 0.0212 | 0.0217 | 0.0222 | 0.0228 |
| **–1.9** | 0.0233 | 0.0239 | 0.0244 | 0.0250 | 0.0256 | 0.0262 | 0.0268 | 0.0274 | 0.0281 | 0.0287 |
| **–1.8** | 0.0294 | 0.0301 | 0.0307 | 0.0314 | 0.0322 | 0.0329 | 0.0336 | 0.0344 | 0.0351 | 0.0359 |
| **–1.7** | 0.0367 | 0.0375 | 0.0384 | 0.0392 | 0.0401 | 0.0409 | 0.0418 | 0.0427 | 0.0436 | 0.0446 |
| **–1.6** | 0.0455 | 0.0465 | 0.0475 | 0.0485 | 0.0495 | 0.0505 | 0.0516 | 0.0526 | 0.0537 | 0.0548 |
| **–1.5** | 0.0559 | 0.0571 | 0.0582 | 0.0594 | 0.0606 | 0.0618 | 0.0630 | 0.0643 | 0.0655 | 0.0668 |
| **–1.4** | 0.0681 | 0.0694 | 0.0708 | 0.0721 | 0.0735 | 0.0749 | 0.0764 | 0.0778 | 0.0793 | 0.0808 |
| **–1.3** | 0.0823 | 0.0838 | 0.0853 | 0.0869 | 0.0885 | 0.0901 | 0.0918 | 0.0934 | 0.0951 | 0.0968 |
| **–1.2** | 0.0985 | 0.1003 | 0.1020 | 0.1038 | 0.1056 | 0.1075 | 0.1093 | 0.1112 | 0.1131 | 0.1151 |
| **–1.1** | 0.1170 | 0.1190 | 0.1210 | 0.1230 | 0.1251 | 0.1271 | 0.1292 | 0.1314 | 0.1335 | 0.1357 |
| **–1.0** | 0.1379 | 0.1401 | 0.1423 | 0.1446 | 0.1469 | 0.1492 | 0.1515 | 0.1539 | 0.1562 | 0.1587 |
| **–0.9** | 0.1611 | 0.1635 | 0.1660 | 0.1685 | 0.1711 | 0.1736 | 0.1762 | 0.1788 | 0.1814 | 0.1841 |
| **–0.8** | 0.1867 | 0.1894 | 0.1922 | 0.1949 | 0.1977 | 0.2005 | 0.2033 | 0.2061 | 0.2090 | 0.2119 |
| **–0.7** | 0.2148 | 0.2177 | 0.2206 | 0.2236 | 0.2266 | 0.2296 | 0.2327 | 0.2358 | 0.2389 | 0.2420 |
| **–0.6** | 0.2451 | 0.2483 | 0.2514 | 0.2546 | 0.2578 | 0.2611 | 0.2643 | 0.2676 | 0.2709 | 0.2743 |
| **–0.5** | 0.2776 | 0.2810 | 0.2843 | 0.2877 | 0.2912 | 0.2946 | 0.2981 | 0.3015 | 0.3050 | 0.3085 |
| **–0.4** | 0.3121 | 0.3156 | 0.3192 | 0.3228 | 0.3264 | 0.3300 | 0.3336 | 0.3372 | 0.3409 | 0.3446 |
| **–0.3** | 0.3483 | 0.3520 | 0.3557 | 0.3594 | 0.3632 | 0.3669 | 0.3707 | 0.3745 | 0.3783 | 0.3821 |
| **–0.2** | 0.3859 | 0.3897 | 0.3936 | 0.3974 | 0.4013 | 0.4052 | 0.4090 | 0.4129 | 0.4168 | 0.4207 |
| **–0.1** | 0.4247 | 0.4286 | 0.4325 | 0.4364 | 0.4404 | 0.4443 | 0.4483 | 0.4522 | 0.4562 | 0.4602 |
| **–0.0** | 0.4641 | 0.4681 | 0.4721 | 0.4761 | 0.4801 | 0.4840 | 0.4880 | 0.4920 | 0.4960 | 0.5000 |

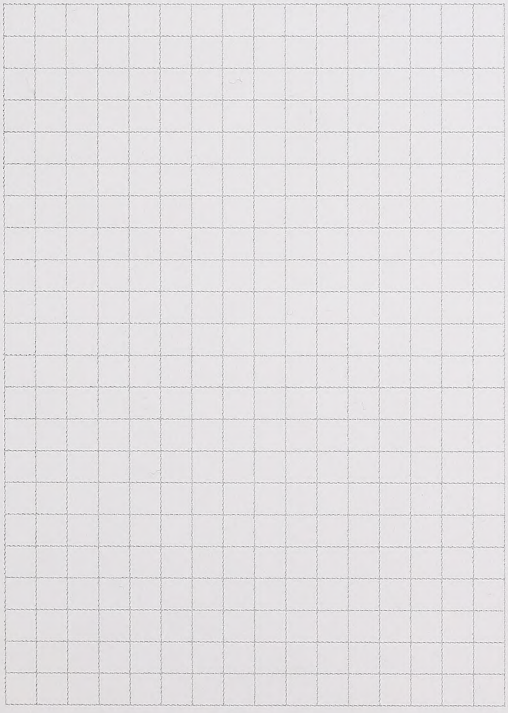
*Fold and tear along perforation.*

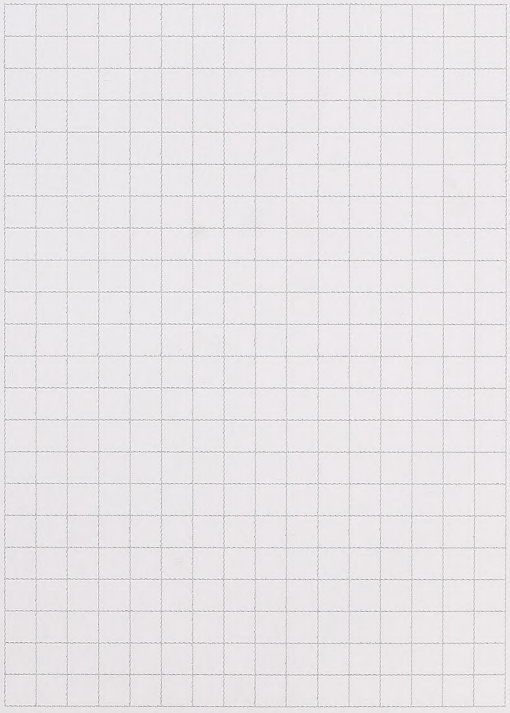
## *Areas under the Standard Normal Curve*

| $z$ | **0.00** | **0.01** | **0.02** | **0.03** | **0.04** | **0.05** | **0.06** | **0.07** | **0.08** | **0.09** |
|---|---|---|---|---|---|---|---|---|---|---|
| **0.0** | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| **0.1** | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| **0.2** | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| **0.3** | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| **0.4** | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| **0.5** | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| **0.6** | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| **0.7** | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| **0.8** | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| **0.9** | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| **1.0** | 0.8413 | .8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| **1.1** | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| **1.2** | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| **1.3** | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| **1.4** | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| **1.5** | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| **1.6** | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| **1.7** | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| **1.8** | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| **1.9** | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| **2.0** | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| **2.1** | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| **2.2** | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| **2.3** | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| **2.4** | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| **2.5** | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| **2.6** | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| **2.7** | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| **2.8** | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| **2.9** | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| **3.0** | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| **3.1** | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| **3.2** | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| **3.3** | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| **3.4** | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |









*Fold and tear along perforation.*

# Pure Mathematics 30

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